Chronic Misapplication of the Relationship Between Magnitude and Frequency in Geomorphic Processes, As Illustrated in *Fluvial Processes in Geomorphology* by Leopold, Wolman and Miller (1964)

Tim Sullivan
USDA Forest Service, Region 1 Water Team, Missoula, Montana

Walt Lucas
USDA Forest Service, R6 Water Resources Team, Klamath Falls, Oregon

M.G. Wolman and J.P. Miller's paper, *Magnitude and frequency of forces in geomorphic processes* (1960), examined the long-term work done by erosion. They showed that for processes like sediment transport, where movement of materials is driven by shear stress, a large proportion of the work is performed by relatively frequent events of moderate magnitude. Erosional work as indexed by the product of the rate of movement of an event and its frequency of occurrence reaches a maximum at a relatively frequent event. The relationship between magnitude and frequency as charted in Figure 3-23 in *Fluvial Processes in Geomorphology* (Leopold et al. 1964) is frequently referenced. Through time the abscissa in this chart has often appeared as "discharge" rather than "applied stress" as originally published. This transposition reasonably defines sediment transport in a channel only for flows up to bankfull depth. However, once flows overtop the bank, the rate of increase for in-channel bed shear (γRS) must show an inflection. This relationship is not shown in the incorrect representations of the magnitude/frequency relation and has implications for methodologies that attempt to calculate total sediment transport at flows in excess of bankfull.

Keywords: *channel geomorphology, bankfull discharge, sediment transport*

Introduction

In their 1960 Journal of Geology paper, *Magnitude and frequency of forces in geomorphic processes*, M. Gordon Wolman and John P. Miller examined the relative importance in geomorphic processes of extreme, so-called catastrophic events and more ordinary events of smaller magnitude. They evaluated the relative amounts of "work" done on the landscape over the long-term by events of all magnitudes. They concluded that a large part of sediment transport by various media is performed by events of moderate magnitude that recur relatively frequently, rather than by rare events of unusual magnitude. Many features of the landscape are formed by frequent events with relatively small applied stress. Notably, and with lasting impact, they found that stream channel dimensions in alluvial channels seem to be molded by flows at or near the bankfull stage.

A conceptual depiction of the relationships discussed in their paper was presented as Figure 1 of the 1960 paper (see our Figure 1). Leopold, Wolman and Miller duplicate that chart as Figure 3-23 in *Fluvial Processes in Geomorphology* (1964). The form of the chart has been frequently duplicated in subsequent papers that purport to reference either that book or the earlier paper.

In many of the subsequent papers, the abscissa of this chart has often been depicted as "discharge," rather than "applied stress" as originally published. We will show a couple of examples, drawn from a target-rich environment, where this has occurred. The fact that discharge and applied stress are not synonymous is obvious and will not be elaborated upon here. Overlooking this distinction has consequences, especially in the estimate of bedload sediment transport at stages above bankfull.

Shear Moves Bedload Sediment

Gravity is the bus that drives sediment to the sea.

There are multiple modes of sediment transport in streams. Bedload is pushed along by shear stress, while suspended and washload sediment must be maintained in suspension by turbulence and upward velocities that equal or exceed settling velocities of particles.

Figure 2 shows the basic vectors of sediment transport in rivers: gravity, shear stress, normal stress, and friction.

M Furniss, C Clifton, and K Ronnenberg, eds., 2007. *Advancing the Fundamental Sciences: Proceedings of the Forest Service National Earth Sciences Conference, San Diego, CA, 18-22 October 2004*, PNW-GTR-689, Portland, OR: U.S. Department of Agriculture, Forest Service, Pacific Northwest Research Station.

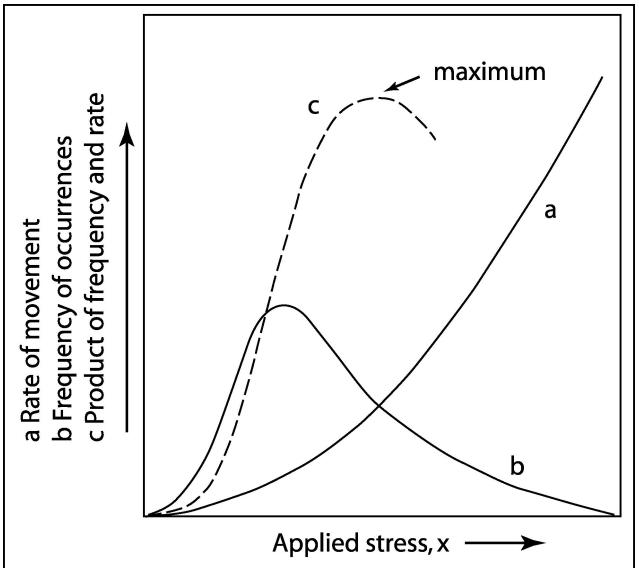

Fig. 1. -- Relations between rate of transport, applied stress, and frequency of stress application.

Figure 1. From **Magnitude and frequency of forces in geomorphic processes***, Wolman and Miller (1960).*

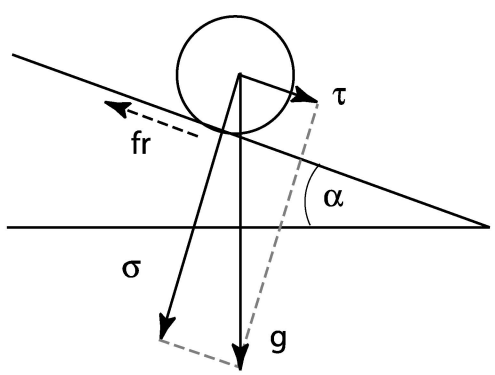

Figure 2. Forces on a particle; bedload sediment transport. Original figure from a no longer available Internet instruction module from the University of Exeter, Department of Geography, Module 1202/2- "Stream Form and Process".

Shear is an applied stress. Commonly denoted by the Greek letter Tau, τ, shear is the force applied tangentially to the sediment surface. It is useful to remember that stress and pressure have the same units: force per unit area, lb/in^2 or N/m^2 .

In *Fluvial Processes*, Leopold, Wolman and Miller describe the relationship between shear and sediment movement in the following terms:

> "To the extent that the movement of sediment by water or air is dependent upon shear stress, the rate of movement can be described by the equation
>
> $$\mathbf{q = k(\tau - \tau^c)^n},$$
>
> where q is the rate of transport, k [is] a constant related to the characteristics of the material transported, τ [is] the shear stress per unit area, τ^c [is] the critical or threshold shear required to move the material, and n [is] an exponent.", p. 79

It is important to remember that shear stress, not discharge, is being described in this equation. The authors' use of the term "q", standard nomenclature for describing discharge, has probably led to some of the later confusion.

This transport relationship is a power function illustrated by the "a" line (Figure 1 in *Magnitude and frequency of forces in geomorphic processes*, also our Figure 1). Applied stress is the independent variable. Figure 3-23 in *Fluvial Processes* is identical. In the text of the 1960 paper and in the subsequent book, Wolman and Miller emphasized that stress must exceed critical threshold values before particles move. Unfortunately, the charts show movement beginning at an implied applied stress just past zero. This, too, has probably led to subsequent confusion.

The Relationship Shown in Wolman and Miller (1960) Has Often Been Misrepresented

Figure 1, the cornerstone chart from *Magnitude and frequency of forces in geomorphic processes*, has been referenced many times. In many instances, the abscissa has been transformed from applied stress to flow. Figures 3 and 4 are two examples.

Flow Increases Faster Than Does Shear at Stages Above Bankfull

All erosional processes are driven by gravity. The gravitational force acting on water drives the flow downslope in a confined channel, where the slope of the water surface represents the balance between the energy gradient and frictional resistance from the channel bed and banks.

Shear and flow are partially products of water depth, both increasing as water stage increases. As water levels exceed bankfull stage, water flows out on the floodplain and flow increases considerably faster than does channel shear. Shear, not flow, is moving bedload sediment.

The transport relationship charted in line "a" of Wolman's Figure 1 remains correct above bankfull stage. Transforming the independent variable from applied stress to flow, as in (our) Figures 3 and 4, leads to an incorrect view of the bedload transport rate at stages above bankfull. Bedload transport continues to increase when flows exceed bankfull, but only as shear increases.

It follows that extrapolation of flow/bedload sediment transport rates determined at less than full channel flows,

Figure 3. Transformation of the abscissa from applied stress to discharge. From Andrews (1980).

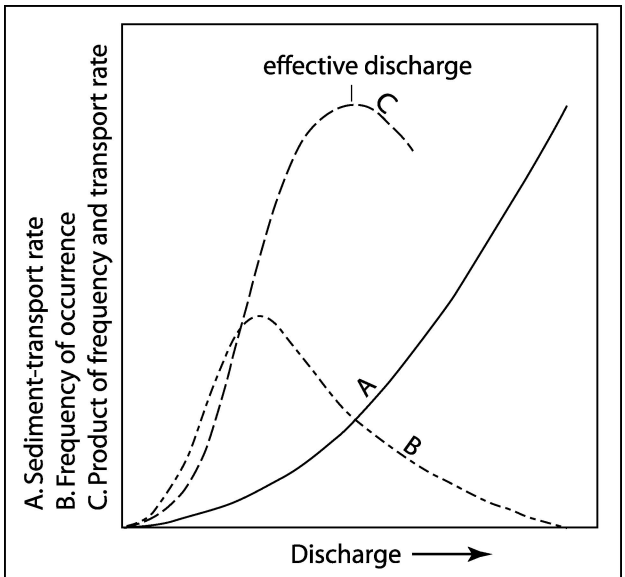

Fig. 1. Relations between discharge and sediment-transport rate, frequency of occurrence, and the product of frequency of occurrence and transport rate (after Wolman and Miller, 1960).

Figure 4. Transformation of the abscissa from applied stress to discharge: A recent example of the same mistake. From Schmidt and Potyondy (2004).

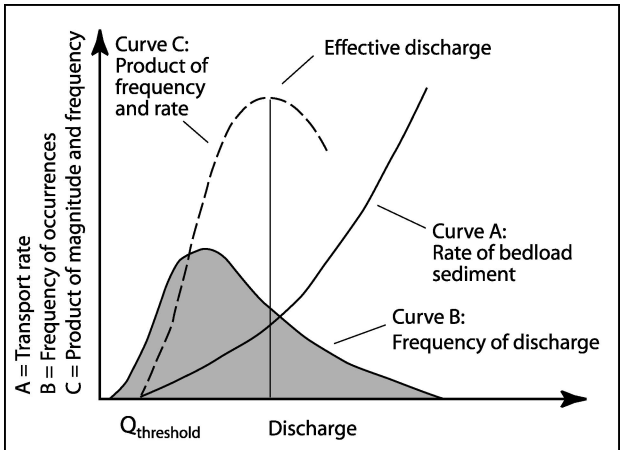

Figure 1 -- The magnitued-frequency concept applied to bed-load transport and disch]arge adapted from Wolman and Miller 1960. The concept suggests that a range of inter-mediate flows transports more sediment over the long-term than either high or low discharges. $Q_{threshold}$ refers to a threshold discharge at which bedload transport begins.

to flows in excess of bankfull, can lead to an overestimation of total bedload transport.

Conclusion

Wolman and Miller's 1960 paper opened innovative lines of inquiry for geomorphology. Figure 1 in that publication has been widely, and often incorrectly, reproduced. The abscissa of that figure is often incorrectly portrayed as "flow" rather than "applied stress" as originally published. Of physical necessity a depiction of bedload sediment as a function of flow needs to show a diminished slope beginning at the stage of bankfull flow.

Literature Cited

Andrews, ED. 1980. Effective and bankfull discharges of streams in the Yampa River basin, Colorado and Wyoming. *Journal of Hydrology* 46:311-330.

Leopold, LB, MG Wolman, and JP Miller. 1964. Fluvial processes in geomorphology. San Francisco, CA: Freeman: 79-80.

Schmidt, LJ, and JP Potyondy. 2004. Quantifying channel maintenance instream flows: an approach for gravel-bed streams in the Western United States. Gen. Tech. Rep. RMRS-GTR-128. Fort Collins, CO: U.S. Department of Agriculture, Forest Service, Rocky Mountain Research Station. 33 p.

Wolman, MG, and JP Miller. 1960. Magnitude and frequency of forces in geomorphic processes. *Journal of Geology* 68(1): 54-74.